EVALUATION OF A PRODUCTION LAYOUT OF A JOINERY IN RIO DE JANEIRO: AN APPROACH IN THE LIGHT OF GRAPH THEORY THROUGH THE CENTRALITY ANALYSIS

STRATEGY IN ORGANIZATIONS

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Abstract

Increasingly, there is a growing effort of organizations focused on competitiveness, aiming to meet the needs of their customers. One of these significant efforts is production layout planning, which is entirely linked to costs, movements, and production time reduction. Planning the distribution of a physical arrangement is an arduous task and requires the use of various techniques, given the losses tied to the possible changes and future corrections, besides that the adoption of an inadequate layout can culminate in inefficient processes, delays, lack of quality, lack of confidence, disorganization and consequently financial losses. The study applies Graph Theory through the analysis of degree centrality, proximity centrality, intermediation centrality, and isomorphism property in the adoption of a new productive layout in a broadly expanding joinery in Rio de Janeiro. Taking the workstations as vertices and their respective interactions as degrees, a broad analysis was performed, where it was possible to identify the workstations with greater importance, enabling the end of this study the proposal to adoption a new layout more appropriate to the current demands of the company.

Keywords: *Operational Research; Graphs; Layout; Centrality Analysis; Physical arrangement.*

1. Introduction

As indicated by SOBRAPO (2021): "Operational Research is the area of knowledge that studies, develops, and applies advanced analytical methods to assist in making better decisions in the most diverse areas of human activity". Taha (2008), states that operational research is composed of several areas of research, some of them: mathematical programming, which can be linear or whole, data science, simulation to discrete events, multicriteria support to decision, queue theory, statistics, data wrapping analysis and Graph theory. Also, according to Taha (2008), operational research can be used in several areas, regardless of the segment. Although large organizations use Operational Research in the quest for problem-solving and obtaining better results, smaller companies and individual entrepreneurs also already use their benefits in solving real problems.

Santos et al. (2017) represent the engineer as a "problem resolver", assigning to him the capability to understand a problematic situation and the formulation of viable solutions, producing value not only for the organization to which he is part but also for society. Rising from a decision problem, the work in question uses graph theory for layout analysis in a largely expanding joinery in Rio de Janeiro, using centrality measures for the evaluation of the most important workstations contained in the production process.

The company in question originally planned and established the distribution of workstations in the physical arrangement of its new warehouse, not counting on the adoption of techniques and specific studies on layout planning, thus adopting a layout that did not contemplate the most appropriate evaluation to maintain a clean flow, without being focused on reducing unnecessary material transport, reduction of employee movement, increase in quality in the processes performed and consequently without directing efforts towards more productivity in the sector.

Throughout the construction of the present work, data were collected from the workstations, their respective interactions, and then the application of graph theory through the resources of measures of degree centrality, proximity centrality, and centrality of intermediation for the evaluation and proposal of a new physical arrangement. To obtain an adequate perception of the size of the problem, a brainstorming was conducted with the managers of the joinery, thus enabling the creation of a mental map (Figure 1) with all relevant relationships on the relevant points in the improvement of the layout.

According to Buzan (2009), the mind map is a dynamic tool that contributes to planning and thinking contribute to activities in a smarter and more agile way, exploring and efficiently utilizing brain resources. "Mind maps help us plan and manage information efficiently and increase the likelihood of success." (BUZAN, T., 2009, p. 11).

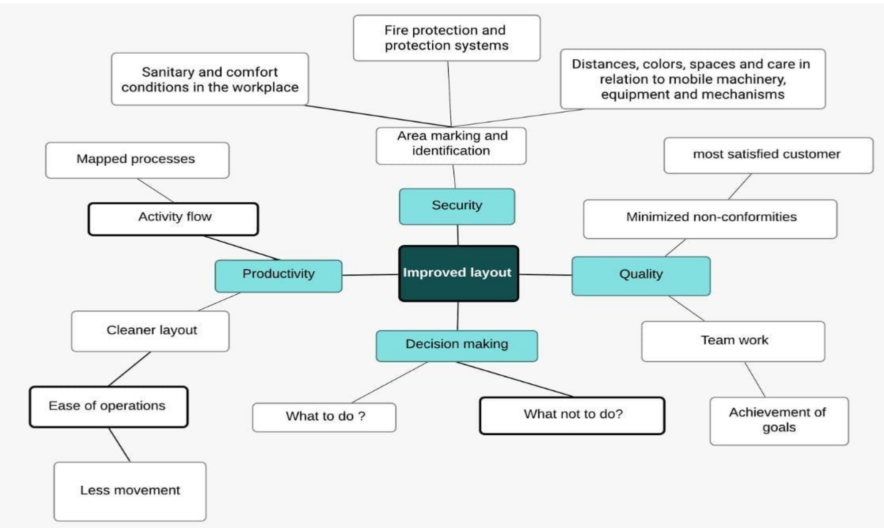

Figure 1: Mind Map - Layout improvements. Source: Authors (2021)

2. PROBLEM

The evaluated company operates in the production of sofas, armchairs, and other various types of furniture in the luxury segment, delivering to the customers super elegant furniture with quality, comfort, and unique designs, all produced based on solid wood. The company will be treated in this study as "Luxury of Furniture", a fictitious name used for reasons of maintaining the confidentiality of the real corporate name of the organization.

Fortunately, the Luxury of Furniture company experiences a moment of great expansion, recounting an increase in demand and consequently, the increase in its production scale, thus culminating in the need to expand its physical space, since the current facilities were no longer sufficient to meet the demand.

It was necessary to rent a new shed, where soon after the places where each activity would be performed were defined, however, a study was not done on the proximity of the activities, that is, where each workstation should be positioned in order to decrease time with unnecessary movements to provide more efficient production and with more fluid flows in production.

The problem of proper layout planning is already a subject of many discussions, and much has been produced about it in the literature. Peinado and Graeml (2004) highlight the importance of decision-making focused on planning and choosing layouts:

> “Physical arrangement decisions define how the company will produce. The layout, or physical arrangement is the most visible and exposed part of any organization. The need to study it exists whenever it is intended to implement a new plant or service unit or when promoting the reformulation of industrial plants or other productive operations already in operation.”
>
> (PEINADO and GRAEML, 2004, p. 200)

For Hermogenes et al. (2020, p. 1), competitiveness is entirely linked to competitiveness: "analyzing and choosing the most appropriate layout is a key point for a company to remain competitive in the market.”

Also according to Peinado and Graeml (2004), the need to make decisions about physical arrangements stems from several reasons, some of them: Need to expand productive capacity; High operational cost and introduction of new product line; Improvement of the work environment and as a complement to Peinado and Graeml (2004) there are five basic principles for an adequate analysis of physical arrangement, being: Safety; Economy of movements; Long-term flexibility; Principle of progressivity and use of space.

Given this, the study in question uses the application of Graph Theory through the concepts of measures and centralities and Isomorphism, since it aims to demonstrate how the current layout should be designed so that production is more efficient, without changing the order and quantity interactions of the processes maintained between workstations.

Measures of centrality of degree, centrality of proximity and centrality of intermediation will be used to score and evaluate the importance of each respective workstation according to their interactions in the productive system, while the concept of Isomorphism will delimit the performance of the workstations, since the ordering and configuration of the processes will be maintained.

3. BIBLIOMETRIC ANALYSIS

To understand the context of this theme in the literature, the following word chain was proposed for investigation through the Scopus database in June 2021: TITLE-ABS-KEY (("Graphs" OR "Graphs Theory" OR "Centrality analysis") AND ("Facility Layout" OR "Production layout" OR "Physical arrangement" OR "Industrial Layout" OR "Layout optimization")), obtaining 142 articles, 63 conference articles, 14 conference review, 3 books and 2 book chapters. With the support of VOSviewer bibliometric analysis software, it was possible to establish the main topics through the use of co-occurrence analysis, as shown in Figure 2:

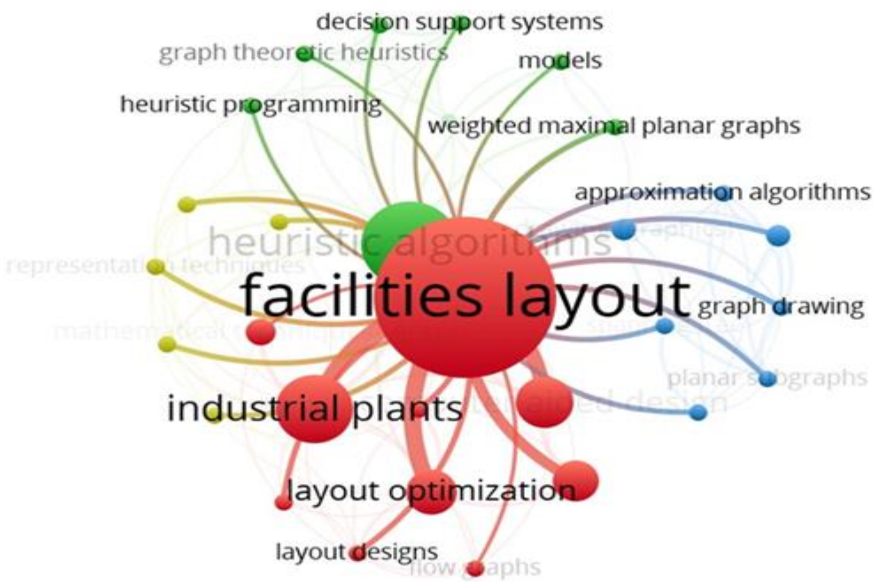

Figure 2: Co-occurrence analysis. Source: Authors (2021)

4. THEORETICAL RATIONALE

4.1 GRAPH THEORY

Diestel (2000) conceptualizes graph theory with an area of mathematics that is in charge of analyzing the relationships maintained between the elements of a given set, defining a graph through the structure G(V,E), being V a non-empty set of nodes and E as a subset of pairs not ordered to V.

Santos et al. (2021) and Del-Vecchio (2009) describe the graph as an extremely powerful tool of Operational Research, commonly used in the construction of models related to engineering, economics, finance, logistics, industry, networks, communication, decision making, etc.

Given its power of analysis in complex problems, countless studies in various areas of activity have been carried out with the support of graph theory, highlighting among them: Finkel et al. (2020) with the analysis of the DOW30 index of the United States through the application of centrality measures, Torres et al. (2016) with the analysis of graph centrality focused on the aerial network of the Alliance Oneworld and Ferreira et al. (2020) with the study of the spread of COVID-19 in a network of students of the Federal Institute of Minas Gerais through the application of centrality measures.

4.2 CENTRALITY MEASURES

The first study related to graph centrality measures was proposed by Bavelas (1950) through a precise and pioneering analysis of social networks and the established communication pattern. Since then much has been done in the light of centrality analyses in various real-world situations, such as in Costa et al. (2015) with the centrality analysis focused on the maintained proximity between airports in the United States; Del-Vecchio et al. (2009) with the analysis of equity investment funds in Brazil, identifying the existing pattern of their connections and influences; Ferreira et al. (2020) with the study of centrality measures in the propagation of covid-19 in a network of university students; and Finkel et al. (2020) with the analysis of the financial market through the application of centrality measures to the Us Dow30 index. Bavelas (1950) conceptualizes the measure of centrality as the measurement of the most important vertex in graph modeling by identifying the "most central" node. Centrality measures can take on a variety of types that depend entirely on the type of analysis desired.

After a broad study of existing centrality measures, Freeman (1978) summarized them in three main concepts: degree centrality, proximity centrality and centrality of intermediation. Then, these definitions were complemented with the centrality of autovector, proposed by Bonacich (1987) and with the centrality of layers, proposed by Bergiante (2004).

To Torres et al. (2016), the main Centrality Measures are conceptualized as:

a) **Degree Centrality Measure** - It takes place by counting existing connections at the vertex. The degree centrality is demonstrated through equation (1), where dk represents the degree of vertex k and aij represent the elements of the adjacency matrix:

$$Cd\ (vk)\ =\ dk\ = \sum_{j=1}^{n} akj \tag{1}$$

b) **Intermediation Centrality Measure** – It is through the frequency that a vertex is contained in the shortest path between two other graph vertices. The intermediation centrality is demonstrated through equation (2), considering i ≠ j ≠ k, and dikj as geodesic paths (shortest path between two other vertices of the graph), maintained between i and j and which have path by k and dij:

$$Cd\ (vk)\ = \frac{\sum dikj}{\sum dij}$$

(2)

c) **Proximity Centrality Measure** - It is through the inverse of the sum of distances maintained between the vertex all other vertices existing in the graph. The centrality of proximity is demonstrated through equation (3), considering dist (vj, vk) as the shortest distance between the vertex vj and vk.

$$Cc\ (vk) = \frac{1}{\sum_{j=1}^{n} dist\ (vj,vk)}$$

(3)

4.3 ISOMORPHISM

The property of Isomorphism was proposed by Whitney (1932), through the creation of Whitney's graph isomorphism theorem. According to Whitney (1932) isomorphic graphs, in addition to the theme of study of classical mathematics, are also targets of complex studies such as in mathematics in the identification of chemical compounds and in electronic automation with the analysis of equivalence in the representations of electronic circuit drawings.

According to Diestel (2000), graph isomorphism can be represented as a bijector of graphs G and H on the vertices G and H, that is, Isomorphism represents a relationship of equivalence between two graphs, which although they can present two apparently different graph representations, retain the same structure.

Diestel (2000) exemplifies the relationship of isomorphism through the following graphs portraited in Figure 3:

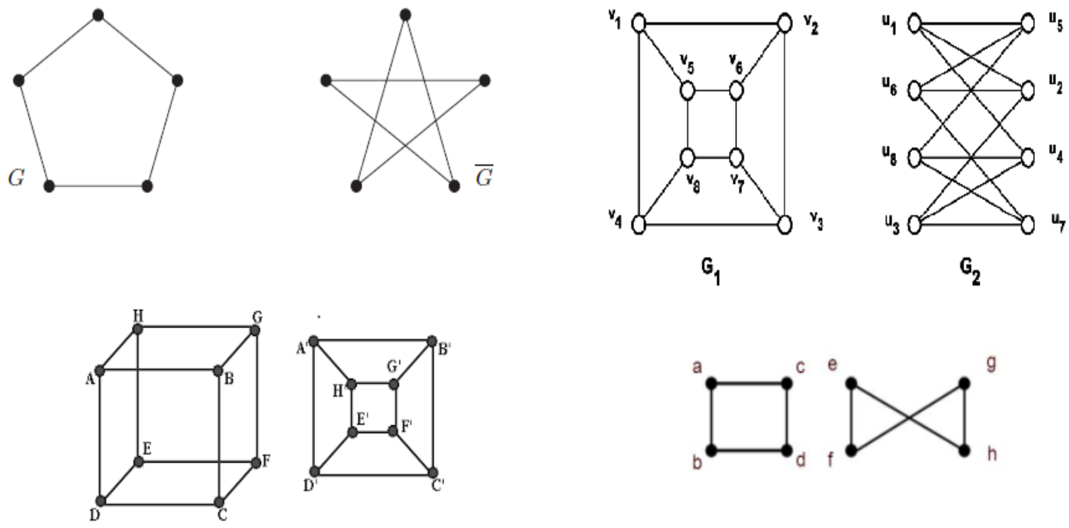

Figure 3: Some isomorphic graphs examples. Source: Adapted from Diestel (2000)

4.4 LEAN MANUFACTURING

According to Womack et al (1990), the Lean Manufacturing production methodology, initially called Toyota's Production System, was created by Toyota in 1950 and proposes the adoption of the lean mindset by the entire company, which consists of reducing waste, pursuing the following organizational objectives: Cost reduction, Continuous improvement , Agility of production, Increase of production capacity and Improvement of the work environment; under the light of the following values: Value, Value Stream, Continuous Flow, Pulled Production and Perfection. (WOMACK et al., 2004).

Womack et al. (2004) summarize the adoption of Lean Manufacturing as the prioritization of organizational efforts in the search for increased production and the reduction of waste; aiming at minimizing costs linked to the human workforce, the use of machinery and equipment, the time and space required for production.

Slack et al. (2009) and Schenk et al. (2010) agree that the choice of the appropriate layout is entirely related to the productivity of organizations, since it acts directly on production time, space saving, minimizing transportation and internal movement, in the labor demanded and in other varied production costs.

Many studies aimed at choosing appropriate layouts were carried out aiming at improving the productive space, highlighting among them: Pattanaik and Sharma (2009) with the application of cellular layouts on the factory floor of Alcast, supplier of armament components in the Indian market, Naqvi et al. (2016) with the application of systematic layout planning in a manufacturing multinational and Hermogenes et al. (2020) with the use of the Analytic Hierarchy Process (AHP) method in the evaluation of production layouts.

4.5 PHYSICAL ARRANGEMENT

According to Slack et. al (2009), the physical disposition of an operation or process translates how its transforming resources are mutually positioned and how the various tasks of the operation are allocated to these transforming resources. According to Santos, Gohr and Laitano, (2012), the impact of small improvements in physical production arrangements is usually directly reflected in the evaluation of business performance, thus improving the organization's productivity indicators.

To Slack et al. (2009), the most practical physical arrangements are derived from only four types, which are: Fixed position physical arrangement; Functional physical arrangement; Cellular physical arrangement; Physical arrangement based on products, or a product line. According to Peinado and Graeml (2004, p. 198): "the mixed arrangement consists of the joint use of two or more physical arrangements.”

As shown in Figure 4, it is possible to see how machines, equipment and employees can be allocated in cellular layouts, by process or on line. In the physical arrangement by fixed position the transforming resources move around the transformed resources, that is, as an example one can cite the construction of a building, such layout according to Peinado and Graeml (2004, p. 228)", also known as positional physical arrangement is the one in which the product, that is, the material to be transformed, remains stationary in a determination of position and transformation resources move around it.”

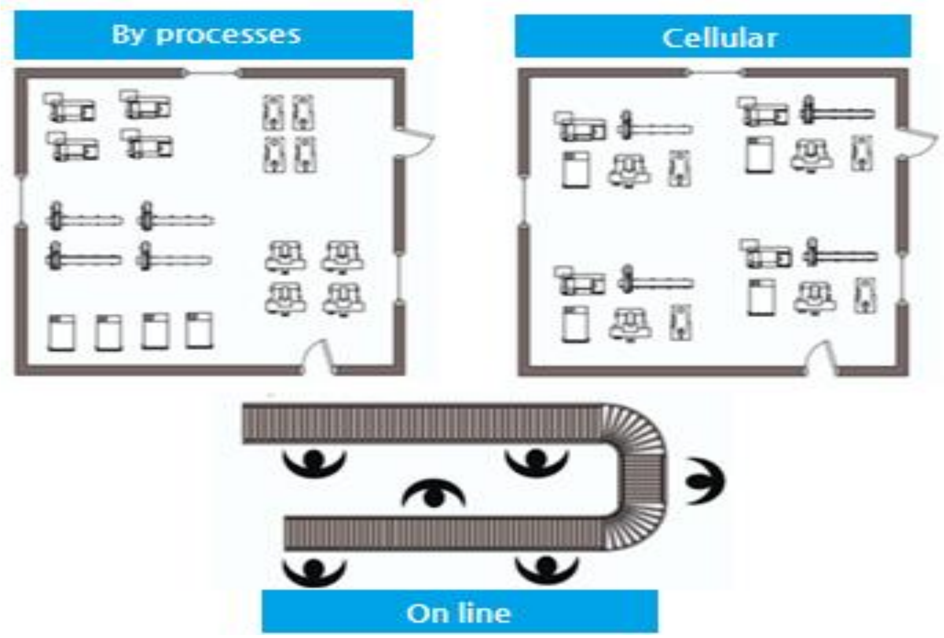

Figure 4: Physical arrangements. Source: Adapted from Peinado and Graeml (2004)

5 CASE STUDY

During the elaboration of the process relationship matrix of workstations and the graph of interactions, it was necessary to map the current layout and investigate the relationship of processes maintained between all sectors, seeking to cover all the relationships of the system as a whole. The data were collected after extensive research applied by the authors directly on the "shop floor" of the Luxury Furniture company.

The Figure 5 indicates the current physical arrangement, where the expansion of activities was carried out with the acquisition of the new shed, not counting the previous study of layout planning, not contemplating a detailed evaluation of the processes and activities that would be carried out and how they would interact with each other, so that it was possible to identify which were the most important workstations in the system:

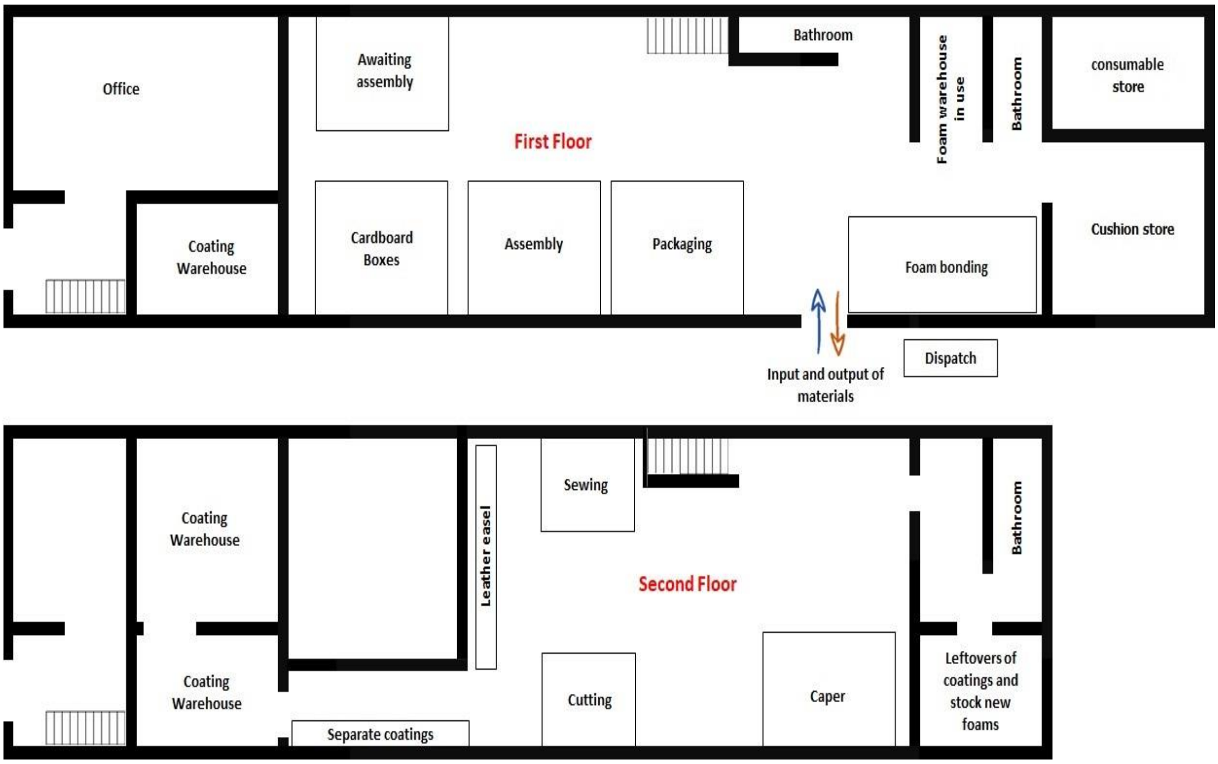

Figure 5: Current Layout. Source: Authors (2021)

During the analysis, the relationship of processes maintained between the following work stations was established: Office (OFF), Coating Warehouse (CW), Awaiting assembly (AA), Cardboard Boxes (CB), Assembly (ASSE), Packaging (PAC), Foam bonding (FB), Foam warehouse in use (FWIU), Cushion Store (CS), Consumable Store (CONSS), Separate coatings (SC), Leather easel (LE) , Cutting (CUTT), Sewing (SEW), Caper (CAP), Leftovers of coatings and stock new foams (LOCASNF) and Dispatch (DISP), as illustrated in Table 1:

	OFF	CW	AA	CB	ASSE	PAC	FB	FWIU	CS	CONSS	SC	LE	CUTT	SEW	CAP	LOCASNF	DISP
OFF	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
CW	0	-	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
AA	0	0	-	0	1	0	0	0	0	0	0	0	0	0	1	0	0
CB	0	0	0	-	0	1	0	0	0	0	0	0	0	0	0	0	0
ASSE	0	0	1	0	-	1	0	0	1	1	0	0	0	0	0	0	0
PAC	0	0	0	1	1	-	0	0	0	1	0	0	0	0	0	0	1
FB	0	0	0	0	0	0	-	1	0	1	0	0	0	0	0	1	0
FWIU	0	0	0	0	0	0	1	-	0	0	0	0	0	0	0	0	0
CS	0	0	0	0	1	0	0	0	-	0	0	0	0	0	0	0	0
CONSS	0	0	0	0	1	1	1	0	0	-	0	0	0	0	0	0	0
SC	0	1	0	0	0	0	0	0	0	0	-	1	1	0	0	0	0
LE	0	0	0	0	0	0	0	0	0	0	1	-	1	0	0	0	0
CUTT	0	0	0	0	0	0	0	0	0	0	1	1	-	1	0	1	0
SEW	0	0	0	0	0	0	0	0	0	0	0	0	1	-	1	0	0
CAP	0	0	1	0	0	0	0	0	0	0	0	0	0	1	-	0	0
LOCASNF	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	-	0
DISP	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	-

Table 1: Relationship of workstation processes. Source: Authors (2021)

After identifying all the relationships between workstations, and evaluating the connection of all vertices, it was possible to generate a graph with all the relationships observed in the layout, as shown in Figure 6 and from these relationships the new layout was generated:

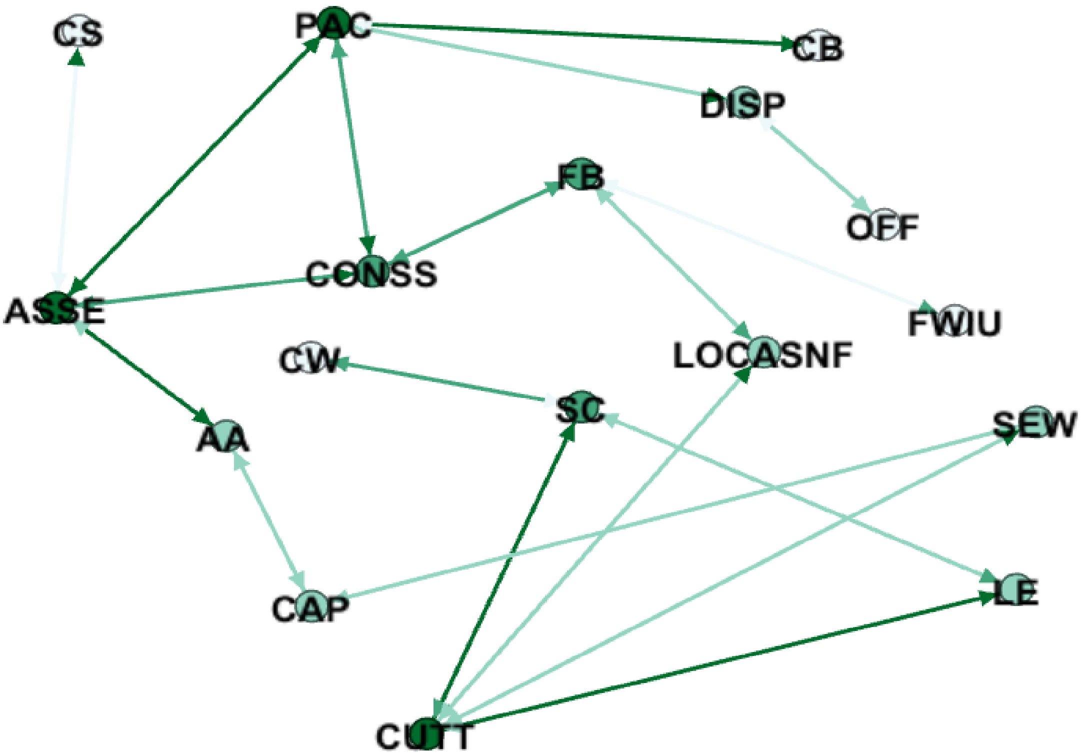

Figure 6: Production Layout Graph. Source: Authors (2021)

With the graph visually arranged, the analysis of centrality of degrees, centrality of proximity and centrality of intermediation was performed with the Gephi graphs software aid, as indicated in Table 2, where it is clearer to observe that the activities involving assembly, packaging and cutting present the greatest centrality of degrees while Assembly, Foam bonding and Consumable Store present the greatest centrality of proximity and Foam bonding, Consumable store and Cutting present the greatest centrality of intermediation.

Setor	Description	Centrality of degrees	Centrality of Proximity	Centrality of intermediation
OFF	Office	1,000	0,219	0,000
CW	Coating Warehouse	1,000	0,213	0,000
AA	Awaiting assembly	2,000	0,340	0,154
CB	Cardboard Boxes	1,000	0,267	0,000
ASSE	Assembly	4,000	0,364	0,275

PAC	Packaging	4,000	0,356	0,342
FB	Foam bonding	3,000	0,381	0,392
FWIU	Foam warehouse in use	1,000	0,281	0,000
CS	Cushion store	1,000	0,271	0,000
CONSS	Consumable Store	3,000	0,390	0,345
SC	Separate coatings	3,000	0,267	0,125
LE	Leather easel	2,000	0,262	0,000
CUTT	Cutting	4,000	0,333	0,375
SEW	Sewing	2,000	0,314	0,121
CAP	Caper	2,000	0,320	0,133
LOCASNF	Leftovers of coatings and stock new foams	2,000	0,356	0,296
DISP	Dispatch	2,000	0,276	0,125

Table 2: Centrality Analysis. Source: Authors (2021)

6 RESULT ANALYSIS

Given the scores obtained through the application of the centrality measures, it was observed that the Assembly, Packaging, and Cutting sectors presented the highest degree of centrality, meaning that they have the highest scores of interactions with the other sectors; the Assembly, Foam bonding and Consumable store sectors presented the highest scores for proximity centrality, presenting greater relative proximity between the other work stations arranged in the system while the Foam bonding, Consumable store, and Cutting sectors presented the highest scores for the centrality of intermediation.

Aligning the study of centrality measures with the particularities and strategies already established in the line and production of the company Luxury of furniture, it was understood that the degree centrality had a greater weight of relevance among the other analyses, being used in the proposal of distribution of workstations in the physical arrangement, pointing out changes that further the arrangement of Assembly stores, Packaging, and Cutting, such a way that it is strategically easy to access them and for the production flows in these workstations to happen more productively.

With the gathered analysis properly structured, a new layout was indicated, maintaining the relationships between the vertices and with different storage to provide more safety, productivity, quality and improve decision-making and virtue of a more fluid arrangement, as shown in Figure 7:

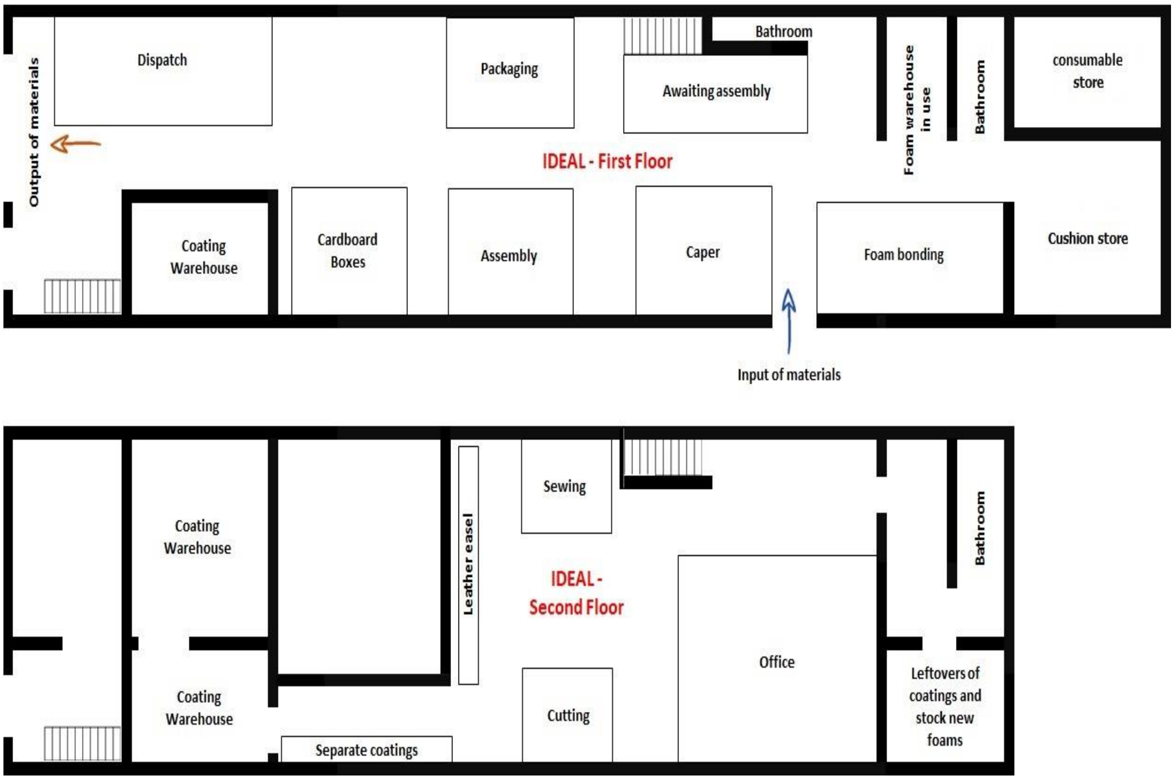

Figure 7: New layout proposal. Source: Authors (2021).

7 CONCLUSION

The graph theory applied in the evaluation of layouts can be of great value for decision making and improvement in the flow of operations in the physical space analyzed since by observing the layout as a network of interconnected activities, it is possible to analyze and generate results in order to improve the productivity of the environment. By applying the concepts of degrees, centrality, and isomorphism, it was possible to indicate new storage of physical space, maintaining all the necessary relationships in the work centers, where this new proposal allows a cleaner, continuous flow with fewer movements, thus corroborating the principles of Lean Manufacturing, besides providing greater quality and safety.

Therefore, it is concluded that the current study goal was achieved, supporting decision-making in a real layout's changing problem faced by the Luxury of Furniture, providing through the analysis of centrality and isomorphism property a robust proposal of arrangement suitable to the company's policies.

In the future, if other organizations wish to implement any innovation in their physical space, the methodology presented in this study can be used as a reference, since the proposal concluded precisely that the physical space could be better used if the proper layout planning was carried out.

Based on the knowledge acquired during the present work, as proposed for future work, it is suggested a combined application between centrality measures and methods to support multicriteria decision-making that can prioritize and/or ponder the variables most relevant to the theme of layout planning.

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